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Utility Theory Application: Insurance Premium (Car Insurance)

If people could predict the consequences of their decisions at the first place, their life would be simpler, but less interesting. Each person makes decisions based on the preferences that contribute to some consequences. However, people do not have a gift of an absolute foresight. The best outcome can be the action, which lead them to the same set of uncertainties, but not to another. A detailed theory has been developed, which helps to make the right decision in the face of uncertainty. This area of knowledge is called a utility theory. Due to the great importance of this theory for insurance systems, it is possible to provide an overview of its main provisions.

One of the approaches to solving the problem of decision making in the face of uncertainty is to determine the value of economic project with a random outcome presented as its average expected value. According to this principle, in terms of decision making process, the expected value, the distribution of possible outcomes can be replaced by a single number, the expected value of a random outcome or expressed. According to this principle, a decision maker should be indifferent to accept incidental loss X or pay the value of E(X) in order to protect themselves from possible losses. Similarly, a decision maker must accept paying the amount not exceeding the quantity of E(Y), in order to participate in high-risk venture with random payments Y. In economics, the expected value of random events, which are involved in cash payments, is often called fair or actuarial value of this event.

Many of the decision makers do not recognize the principle of the expected value. They believe that their decisions affect the size of their capital, as well as other characteristics of the distribution of the outcomes.

The starting point of the theory is the assumption that a reasonable person, faced with two distributions of the outcomes that affect the capital, will be able to express a preference for either one of these distributions, or the same attitude towards both assumptions. The preferences must satisfy certain requirements of concurrence. The highest point of the theory is the theorem, which states that if the preferences satisfy the requirements of concurrence, there is a utility function u (w) stated that if the distribution of X is preferred over the distribution of Y, then E [u(X)] > E [u(Y)], and, if the decision maker does not give the preference to any of these distributions, then E [u(X)] = E [u(Y)]. Thus, a qualitative preference of a lack of it can be replaced by comparing numbers.

Before discussing the applications of the utility theory in insurance, it is possible to outline a number of allegations regarding the utility. First, the utility theory is based on the assumption of the existence and consistency of preferences over distributions of probability of possible outcome. Utility function should not reflect any surprises. It is a numerical description of existing preferences. Second, the utility function should not be determined uniquely. For example, if $u^*(w)$ =

au(w) + b, when a > 0, the ratio E [u(X)] = E [u(Y)] is equivalent to E [u*(X)] = E [u*(Y)]. Thus, the preferences are saved, if the utility function is a linear transformation of the original one preference with positive coefficients. Third, it is possible to assume that the utility function is linear and the angle of inclination is positive, which is u (w) = aw + b with a > 0. In this case, if E [X] = μx and E [Y] = μy, then E [u (X)] = aμx + b > E [u (Y)] = aμy + b, but only if μx > μy. Thus, the preferences of distribution of outcomes for an increasing linear utility function are ordered the same way as the expectations of these distributions. Consequently, if the utility function is linear and increasing, then the principle of the expected values for rational economic behavior in the face of uncertainty is not contrary to the rule of the expected utility.

When discussing a car insurance area, it is relevant to apply the utility theory to a decision making problem faced by a person whose property is at risk. Such owner of property has a utility function of capital u (w), where the capital of w is measured in monetary units. It may incur losses due to the onset of random events that may harm the property. The distribution of random losses X is assumed to be known. The owner of the property is indifferent whether to pay the sum of G to the insurer, shifting this person occasional financial loss, or take a risk on himself/herself. This situation can be formalized with the next proportion, such as:

$$u(w - G) = E[u(w - X)].$$

The right-hand member of the equation is the expected utility in case of failure of the purchase of the insurance contract, if the current capital value of the owner of the property is equaled to w. The left-hand side is the expected utility in the payment of G for a full financial coverage. If the owner's utility function is linear and increasing, i.e. $u(w) = bw + d$, $b > 0$, then it will take the principle of the expected value. In this case, the owner of the property or prefers any of the features or favors insurance on condition when:

$u(w - G) - b(w - G) + d \geq E[u(w - X)] = E[b(w - X) + d]$,
$b(w - G) + d \geq b(w - \mu) + d$, $G \leq \mu$.

Thus, if the owner's utility function is increasing and linear, the premium at which it does not give preference to any of the facilities or prefer to purchase full insurance coverage, does not exceed the expected losses. In the absence of subsidies for a long-tem planning, the insurer must ensure that the premiums received have exceeded the expected losses. Therefore, in this case the conclusion of the insurance contract is mutually unlikely. When executing the insurance contract, the insurer must to appoint a premium that is larger than the expected damages and costs in order to avoid a failure of receipts. Thus, the utility function of the owner of the property cannot be liner.

For example, a community has been offered a program for car theft by an insurance company. There are 1,000 families consisting in this community, every of them owns a car evaluated in $10,000. Thus, the program is a proposition for every member of the community. In case the car is stolen, the insurance premium will equaled to $10,000 including no costs. There is 1% per year of the chance a car can be stolen, which may cost the insurance company the compensation of $100,000 per year of coverage for all of the cars. When estimating the average amount of the cost, which is estimated as $100,000 coverage per year, divided to 1,000 car owners in the community, the expenses would be $100 per year that is 1% of the car value, excluding indirect costs. Thus, the fair premium, which is the expected value of the loss, is $100 per year. Taking to the account the given figures, it is possible to estimate the expected value, which is:

EV (including insurance) = 100% * ($10,000 - $100) = $9,900.

Thus, it is possible to outline that the expected value will be almost the same with minimum losses in case if the $10,000 car is lost. In order to realize whether the deal is reasonable, it is necessary to discuss the situation excluding insurance. Hence,

EV (without insurance) = 99% * ($10,000 – 0) + 1% * ($10,000 - $10,000) = $9,900.

The discussion of both situations allows getting to the conclusion that there is no difference in the expected value. Thus, the deal of acquiring insurance is reasonable for a car owner.

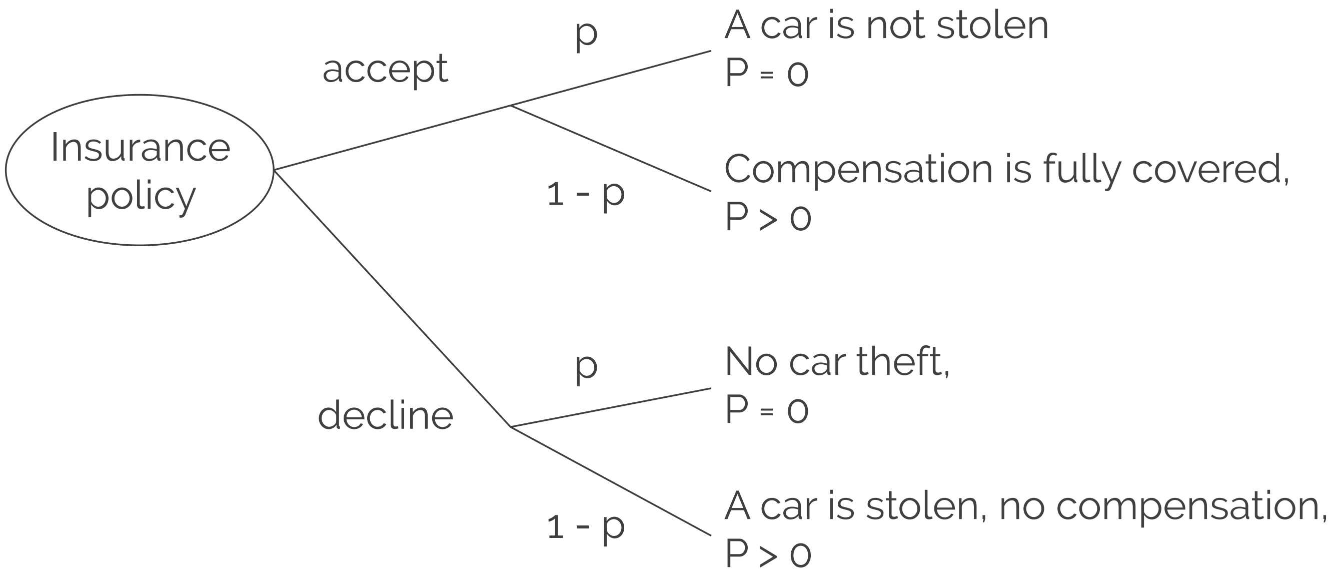
Considering the same deal in terms of utility, lets establish the utility function where U ($x) = In ($x). Here, the utility function is:

EU (insurance) = U [100% * ($10,000 - $100)] = In (9,900) = 9.20

EU = U [99% * ($10,000 – 0)] + U [1% * ($10,000 - $10,000)] = 0.99 In (9,900) = 9.11

To sum up, acquiring insurance would contribute to a higher expected utility rather than not doing it with the ratio of 9.20 and 9.11 respectively (Steinemann).

A decision tree for utility estimations can be presented in the figure 1



where p – the chance of car theft per year (%)

Fig. 1. Decision tree for utility estimations.

When talking about the usage of the application in car insurance area, I would try to use it in order to see the options I might have. However, there is a difficulty in mathematical calculations, which may prevent obtaining a correct data. Thus, the decision making process can become difficult for the user in terms of accepting the final decision.

Works Cited

Steinemann, Ann C. *Macroeconomics for Public Decisions*. 2nd ed. Menlo Park, CA: Askmar Publishing, 2011. Print.